



University of Groningen

The negative impact of noise on the performance of portfolio managers and the need to measure it

Plantinga, Auke

IMPORTANT NOTE: You are advised to consult the publisher's version (publisher's PDF) if you wish to cite from it. Please check the document version below.

Document Version

Publisher's PDF, also known as Version of record

Publication date:

2004

[Link to publication in University of Groningen/UMCG research database](#)

Citation for published version (APA):

Plantinga, A. (2004). The negative impact of noise on the performance of portfolio managers and the need to measure it. s.n.

Copyright

Other than for strictly personal use, it is not permitted to download or to forward/distribute the text or part of it without the consent of the author(s) and/or copyright holder(s), unless the work is under an open content license (like Creative Commons).

Take-down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

Downloaded from the University of Groningen/UMCG research database (Pure): <http://www.rug.nl/research/portal>. For technical reasons the number of authors shown on this cover page is limited to 10 maximum.

The negative impact of noise on the performance of portfolio managers and the need to measure it

April 2004

Auke Plantinga^{*}

1. Introduction

Active portfolio management involves trading on forecasts of the future returns of securities. The level of trading actually going on in investment portfolios suggests large differences between portfolio managers, which indicate differences in forecasting abilities. In this paper, we focus on the quality of the forecasting abilities by making a distinction between information based trading and noise trading. Most performance measures used in the literature ignore this noise-trading component. We show that noise trading is harmful to the investor, and we propose to measure noise trading in addition to the usual measures of performance. We develop alternative measures of noise trading that can be used in empirical research.

Although investment managers increasingly adopt the lessons of modern portfolio theory, it is clear that the majority of the market participants are still engaged in active investing. There is no reason to suspect that this will change, since efficient markets require information-based investors who trade to exploit deviations from market efficiency. However, Black [1986] argues that markets with only information-based investors cannot exist and he believes that noise traders are necessary in order to facilitate a liquid market. De Long et al. [1990] use a simple overlapping generations model with one risky asset to show that noise traders generate additional risk to a market, which allows them to earn a risk premium to remain in the market. In other words, the function of noise trading to provide liquidity in a financial market is rewarded.

^{*} Auke Plantinga is associate professor of finance at the University of Groningen, P.O. Box 800, 9700 AV Groningen, The Netherlands. This paper benefited from help by Nanne Brunia, Frans Tempelaar and Jaap Wierenga, and from suggestions forwarded by participants in a Theme E Seminar of the SOM research school of the University of Groningen.

Noise trading in the sense of De Long et al. [1990] refers to information relating to the ‘true’ value of the risky asset. There are numerous data sources available that may reveal information on the true value of risky assets. However, in practice it may be difficult for active investors to separate relevant information from irrelevant information regarding the true value of a risky asset¹. Therefore it is likely that investors may trade on noise. In this paper we develop methods to measure the degree of noise trading in a portfolio. We define a noisy signal n as a signal that does not bear any relation with future return realizations r :

$$\text{cov}(r, n) = 0. \quad (1)$$

Our measure of noise is different from that of De Long et al., since they focus on a definition of noise relative to the fundamental or true value of a security and our measure refers to the future price of the security, which may be affected by the behavior of noise traders. Knowing that the market price of a stock is affected both by its fundamental value and the behavior of noise traders, successful portfolio management also involves the forecasting of the future behavior of these noise traders.

In this paper we analyze the impact of noisy signals on performance measures. We extend earlier work by for example, Dybvig and Ross [1985] and Grinblatt and Titman [1989,1993]. This earlier work focused on the ability of performance measures to detect forecasting abilities of portfolio managers in the presence of informative signals. Within the context of the Dybvig and Ross model, we show that the standard deviation of portfolio weights is also a measure of the forecasting skills of a portfolio manager. Furthermore, we propose a new measure, the so-called signal measure that measures the forecasting skills of portfolio managers irrespective of their risk aversion. Next, we introduce portfolio managers that act also on noisy signals. This analysis is based on an extended version of the Dybvig and Ross [1985] model of asymmetric information. We show the impact of noisy signal on the outcomes of performance measures. Furthermore, we develop measures explicitly focusing on the detection of noise in portfolio.

The paper is structured as follow. First we present the model of Dybvig and Ross and some of their results on performance measures. Next, we extend their model by allowing for noisy signals. Based on this extended model we provide some insights on the potential damage of noise trading on a portfolio’s performance as well as the consequences for Jensen’s alpha. We investigate methods for measuring the degree of noise trading based on empirical data and propose a method for measuring the level of informative trading and noise trading in a portfolio.

¹ The use of the term ‘irrelevant information’ is of course a contradiction in terms.

2. Signals without noise

Dybvig and Ross [1985] use a model of asymmetric information to study the ability of performance measures to detect managers with forecasting abilities. The basic idea in their model is that the informed manager can predict a part of the future return of a security. An important characteristic of this model is that the manager receives valuable signals that are not affected by noise. In this model, the return from the perspective of an uninformed manager is:

$$r = p + e_u \quad (2)$$

where p is the risk premium and e_u is the uncertainty regarding the normally distributed returns of the risky asset with expectation zero and variance $\sigma_{e_u}^2$. From the perspective of the informed manager, the return on a security is equal to

$$r = p + s + e_i, \quad (3)$$

where s reflects the signal received by the manager. The signal s has a normal distribution with expectation zero and variance σ_s^2 . This signal allows him to adjust his expected return. The informed investor differs from the uninformed investor with respect to his expectations regarding the risky asset. From the perspective of the informed investor, the expected return on a security is $p + s$ and the risk on a security is equal to $\sigma_{e_i}^2 = \sigma_{e_u}^2 - \sigma_s^2$. This results in the following allocation to the risky security:

$$x = \frac{p + s}{A\sigma_e^2}. \quad (4)$$

The expected return on this portfolio as observed by an outside evaluator is equal to

$$E[xr] = \frac{p^2 + \sigma_s^2}{A\sigma_{e_i}^2}, \quad (5)$$

and its variance is equal to:

$$\frac{4p^2\sigma_s^2 + 2\sigma_s^4 + (p^2 + \sigma_s^2)\sigma_{e_i}^2}{\sigma_{e_i}^4 A^2}. \quad (6)$$

Dybvig and Ross used this model to investigate the properties of risk-adjusted performance measures. They derived the following expression for Jensen's alpha:

$$J_a = \frac{\sigma_s^2(\sigma_{e_i}^2 + \sigma_s^2 - p^2)}{A\sigma_{e_i}^2(\sigma_{e_i}^2 + \sigma_s^2)} = \frac{\sigma_s^2}{A\sigma_{e_i}^2} \frac{(\sigma_r^2 - p^2)}{\sigma_r^2} \quad (7)$$

Based on this expression, Dybvig and Ross conclude that Jensen's alpha is inappropriate for evaluating the performance of portfolio managers, since the expression could become negative, even if the manager has forecasting skills. This occurs if the risk premium exceeds the standard deviation of the risky asset. Another observation is that the outcome of Jensen's alpha is determined by the risk aversion of the investor. This means that differences in alphas do not necessarily imply that the managers have different forecasting abilities.

The model of Dybvig and Ross [1985] is rather optimistic on the nature of the forecasting skills of the manager. All the information obtained by the manager is relevant, and the manager will never trade on signal without information. A consequence is that all changes in the manager's portfolio are directly related to his forecasting skills. The measurement problem as identified by Dybvig and Ross can be resolved by observing the standard deviation of the portfolio weights:

$$\sigma_x = \frac{\sigma_s}{A\sigma_e^2}. \quad (8)$$

The standard deviation of portfolio weights highlights the implications of ignoring the existence of noisy signal in the Dybvig and Ross model. Assuming that a manager receives only informative signals implies that all changes in a portfolio are successful, which may tempt us into believing that a manager with a high portfolio turnover is a good manager.

Grinblatt and Titman [1993] proposed to calculate the covariance between portfolio weights and subsequent portfolio returns. They showed that the covariance is also a measure of forecasting abilities:

$$\begin{aligned} \text{cov}(x, r) &= E(xr) - E(x)E(r) \\ &= E\left[\frac{p+s}{A\sigma_e^2}(p+s+e) - \frac{p}{A\sigma_e^2}p\right] = \frac{\sigma_s^2}{A\sigma_e^2} \end{aligned} \quad (9)$$

The expression for the covariance measure is closely related to that of Jensen's alpha.

Jensen's alpha is equal to the covariance measure multiplied by a term $\frac{(\sigma_r^2 - p^2)}{\sigma_r^2}$,

which is always smaller than one, since variances and the risk premium are positive. Compared to the covariance measure, Jensen's alpha tends to underestimate the forecasting abilities.

Since both the standard deviation of portfolio weights and the covariance measure are affected by the risk aversion of the trader, a better measure is the ratio of the

covariance over the standard deviation, which is a direct measure of the standard deviation of the signal:

$$\frac{\sigma_{xr}}{\sigma_x} = \frac{\frac{\sigma_s^2}{A\sigma_e^2}}{\frac{\sigma_s^2}{A\sigma_e^2}} = \sigma_s \quad (10)$$

This measure, which we call the ‘signal measure’ is a truly risk-adjusted measure of the forecasting skills of the portfolio manager.

3. Noisy signals

In order to investigate the consequences of noisy signals, we extend the Dybvig and Ross model by introducing a manager who trades on informative signals as well as noise. We refer to this trader as the informed noise trader. In order to derive a meaningful model, we assume that the informed noise trader is not able to separate the informative signal from the noisy signal.

Following Dybvig and Ross [1985], we assume that the return on the risky security is determined by:

$$r = p + e_u \quad (11)$$

The portfolio manager believes that the return is determined by:

$$r^* = p + s + n + e_i^*, \quad (12)$$

where n is a noisy signal that is normally distributed with expectation 0 and standard deviation σ_n . Since n is noisy, it is uncorrelated with r , although the manager believes otherwise. The signal s is uncorrelated with n . The actual realization will be based on

$$r = p + s + e_i \quad (13)$$

Since the manager is not able to separate n from s he will trade on both signals simultaneously. The changes the manager’s return expectation is

$$E_i^*[r] = p + s + n, \quad (14)$$

and his perception of risk into

$$\sigma_{e_i^*}^2 = \sigma_r^2 - \sigma_s^2 - \sigma_n^2. \quad (15)$$

Based on his perception of expected return and risk, the manager constructs the following portfolio

$$x = \frac{p + s + n}{A\sigma_{e_i}^2}. \quad (16)$$

Given the fact that the signal is a stochastic variable, the expected return of a manager acting on such signals is:

$$E[xr] = \frac{p^2 + \sigma_s^2}{A\sigma_{e_i}^2}. \quad (17)$$

Notice that this is an unconditional expression referring to the expected return as observed by an outside evaluator who cannot observe individual signals. Furthermore, the expression is almost identical to the expected return for the informed trader, with the exception of $\sigma_{e,i}^*$ which is smaller than $\sigma_{e,i}$. As a result, the expected return of the informed trader has a lower return than the informed noise trader with a similar informative signal and similar risk aversion. In other words, the informed noise trader believes that the risky security is less risky than the informed trader does, and therefore he assumes a larger position in the risky asset. In this sense, the informed noise trading is related to overconfidence².

The standard deviation of portfolio weights equals:

$$\sigma_x = \sqrt{\frac{\sigma_s^2 + \sigma_n^2}{A^2\sigma_{e_i}^4}} \quad (18)$$

Since the standard deviation of the portfolio is also determined by the variance of the noisy signal, observing the standard deviation of portfolio weights is no longer enough to conclude that a manager is involved in informed trading. The challenge is to figure out the level of informed trading as compared to noise trading. This is important, as can be observed from the variance of the informed noise trader's portfolio variance.

The variance of the informed noise trader's portfolio equals:

$$E[(xr)^2] - (E[xr])^2 = \frac{2\sigma_s^4 + 4p^2\sigma_s^2 + \sigma_e^2(p^2 + \sigma_s^2 + \sigma_n^2) + \sigma_n^2(p^2 + \sigma_s^2)}{A^2\sigma_{e_i}^4} \quad (19)$$

This expression shows that the noise-trading component is responsible for an increase in the return volatility of the portfolio. The negative impact of noise trading in this model is the result of the increased return volatility of the portfolio. In order to measure the actual utility loss of an investor due to noise trading, it is necessary to

² Overconfidence is usually defined as the tendency of individuals to assign a larger reliability to their own forecasts than actually is justified.

measure the degree of noise trading. However, we might get an impression of the relevance of the problem by constructing an example with some plausible parameters.

Table 1: An illustration of the impact of informative and noisy signals on performance

	Uninformed trader	Informed trader	informed noise trader	noise trader
p	4.0%	4.0%	4.0%	4.0%
σ_r	20.0%	20.0%	20.0%	20.0%
σ_s	0.00%	4.00%	2.83%	0.00%
σ_n	0.00%	0.00%	2.83%	4.00%
σ_e	20.0%	19.6%	19.6%	19.6%
A	2	2	2	2
$E[x_a]$	50%	52%	52%	52%
$E[r_a]^3$	2.0%	4.2%	3.1%	2.1%
$\sigma[r_a]$	10.0%	15.3%	14.9%	14.6%
Sharpe	20.0%	27.2%	21.0%	14.3%
$\sigma[x_a]$	0.0%	52.1%	52.1%	52.1%
Turnover ⁴	0.0%	58.8%	58.8%	58.8%

In table 1, we present an example of the impact of noise trading on portfolio performance. The example involves four different investors, each acting on signals with the same magnitude. The investors are different as the information content of their signals is different. The first column presents an uninformed trader, who basically allocates his wealth to risky assets based on the risk premium. At a risk premium of 4%, a standard deviation of 20% and a constant of absolute risk aversion he allocates 50% of his wealth to the risky asset. His expected portfolio return is 2% with a standard deviation of 20%. Compare this to the informed trader who receives informative signals with a standard deviation of 4%. This trader will generate an additional return of 2.2% with a standard deviation of 15.3%. The corresponding portfolio turnover is equal to 58.8%. The third and the fourth column refer to noise traders who tend to believe that their signals have a standard deviation of 4%, although in reality they also trade on noise. The third column refers to a trader who trades equally on information as on noise, whereas the fourth column refers to a trader who trades on noise only. Obviously, the pure noise trader performs worst. Although his expected performance is slightly higher than that of the uninformed trader, the standard deviation of his portfolio is considerably higher than that of the uninformed trader. The informed noise trader takes a position somewhere in between. Although his expected return is considerable higher than the uninformed trader's expected return, his Sharpe ratio is only slightly higher as compared to that of the uninformed

³ In this table, we present the unconditional expected return and standard deviation as observed by an outside evaluator. The subscript a refers to the results for the actual portfolio.

⁴ Turnover is calculated using the measure derived in the appendix.

trader. This is of course due to the increased volatility as compared to an investor who acts on an equally informative signal without noise.

The model shows that for portfolio with similar portfolio turnover, it is possible to find large differences in performance due to differences in the level of noise present in the signals used by the manager.

4. Measuring informative and noisy signals

In the previous section we showed that noisy signals have a negative effect on the performance of an informed noise trader. In this section, we study the ability of performance measures to deal with noisy signal. First, we investigate the impact of noisy signal on regular performance measures. Next, we study two alternative methods for measuring the degree of noise trading in a portfolio. We face the challenge of deriving measures for the informative and the noisy signal in the context of the model with noisy signals.

4.1 The impact of noisy signals on performance measures

In this section we investigate the impact of informed noise trading on Jensen's alpha, and the signal measure. In order to determine the effect of informed noise trading on the outcome of Jensen's alpha, we start with its definition:

$$J = E[x_a r] - \frac{\text{cov}(x_a r, x_p r)}{\text{var}(x_p r)} E[x_p r] \quad (20)$$

The benchmark for the security market analysis is based on a passive benchmark, which implies a constant weight in the risky asset. It can be shown that the exact allocation to the benchmark portfolio is irrelevant for the outcome of the analysis. The expected return on the benchmark portfolio is

$$E[x_p r] = x_p p$$

The covariance between the return of portfolio a and the benchmark is equal to

$$\begin{aligned}
\text{cov}(x_a r, x_p r) &= E[x_a r x_p r] - E[x_a r]E[x_p r] \\
&= E\left[\frac{p+s+n}{A\sigma_{e_i}^2}(p+s+e_i)x_p(p+s+e_i)\right] - \left(\frac{p^2+\sigma_s^2}{A\sigma_{e_i}^2}\right)x_p p \quad (21) \\
&= \frac{x_p p}{A\sigma_{e_i}^2}(2\sigma_s^2 + \sigma_e^2)
\end{aligned}$$

The variance of the benchmark equals $\text{var}(x_p r) = x_p^2(\sigma_s^2 + \sigma_{e_i}^2)$ so Jensen's alpha becomes:

$$\begin{aligned}
J &= \frac{p^2 + \sigma_s^2}{A\sigma_{e_i}^2} - \frac{\frac{x_p p}{A\sigma_{e_i}^2}(2\sigma_s^2 + \sigma_e^2)}{x_p^2(\sigma_s^2 + \sigma_{e_i}^2)} x_p p, \quad (22) \\
&= \frac{\sigma_s^2(\sigma_s^2 + \sigma_{e_i}^2 - p^2)}{A\sigma_{e_i}^2(\sigma_s^2 + \sigma_{e_i}^2)}
\end{aligned}$$

which is almost identical to the expression for the informed trader. Again the important difference is $\sigma_{e_i}^2 < \sigma_{e_i}^2$. Due to his noisy signal, the manager perceives the risky asset as less risky than is actually justified. The effect is that the manager assumes more risk. So noise trading has the same effect on Jensen's alpha as a lower coefficient of risk aversion.

In section 2 we introduced the signal measure that corrects for the risk attitude of the portfolio manager. The signal measure was defined as the ratio of the covariance between weights and return over the standard deviation of portfolio weights. In the absence of noisy signals the signal measure is an unbiased measure of the forecasting abilities of the portfolio manager. With a noisy signal, the signal measure becomes:

$$\frac{\sigma_{xr}}{\sigma_x} = \frac{\frac{\sigma_s^2}{A\sigma_{e_i}^2}}{\frac{\sqrt{\sigma_s^2 + \sigma_n^2}}{A\sigma_e^2}} = \frac{\sigma_s^2}{\sqrt{\sigma_s^2 + \sigma_n^2}} = \frac{\sigma_s}{\sqrt{1 + \gamma^2}} \quad (23)$$

where $\gamma = \sigma_n / \sigma_s$. Since $\gamma > 0$, the signal measure tends have a downward bias that is increasing with the level of noise trading in the portfolio. The relevance of these biases are illustrated in table 2, where we calculated the explained variance and the residual variance.

In table 2 we present the outcomes for these measures that refer to the same example used for table 1. We compare the informed noise trader from table 1 with a new trader, which is an informed trader that acts on an informative signal only comparable to the informative signal of the informed noise trader. Comparing the outcomes of all

the performance measures for these two traders, we observe that the informed noise trader scores better on expected return, Jensen's alpha and the covariance measure. These performance measures reward the noisy signal, even though noise reduces the expected utility derived from those measures. Notable exception is the signal measure, which is negatively affected by the level of noise.

Table 2: The impact of noisy signals on performance measures

		(1) informed trader	(2) informed noise trader
Parameters	μ	4.00%	4.00%
	σ_r	20.00%	20.00%
	σ_s	2.83%	2.83%
	σ_n	0.00%	2.83%
	σ_e	19.80%	19.60%
Estimators	$E[x]$	51.02%	52.08%
	$E[r]$	3.06%	3.13%
	$\sigma[r_a]$	12.79%	14.91%
	$\sigma[x_a]$	36.08%	52.08%
	Jensen	0.98%	1.00%
	Covar	1.02%	1.04%
	Signal	2.83%	2.00%
	β_a	52.04%	53.15%

Depending on the objective of the measurement it is possible to draw different conclusions. If the objective is to have an unbiased measure of the signal, none of these measures suffice, unless the observer can be sure that the portfolio manager acts on informative signals only.

4.2 Measuring noise trading with linear regression analysis

An intuitive approach in separating the informative trading from noise trading is to estimate the following linear regression model:

$$x = \alpha + \beta_x r + \varepsilon, \quad (24)$$

where x is a vector of portfolio weights, and r is vector of security returns. In other words, we analyze the variance in portfolio weights and explain those by changed in future returns. Then we associate informed trading with those variances in portfolio weights explained by the future returns ($\beta^2 \sigma_r^2$) and the unexplained variance in portfolio weights to noise trading (σ_ε^2). In the remainder of this section we check whether this is a meaningful approach. First, we derive an expression for β_x :

$$\beta_x = \frac{\sigma_{xr}}{\sigma_r^2} \quad (25)$$

The covariance between portfolio weights and subsequent security returns is equal to

$$\sigma_{xr} = E \left[\frac{p+s+n}{A\sigma_{e_i}^2} (p+s+e_i^*) - \frac{p}{A\sigma_e^2} \pi \right] = \frac{\sigma_s^2}{A\sigma_{e_i}^2}. \quad (26)$$

Noise trading does not significantly alter the covariance measure as compared to a model without noisy signals. It is still a measure of the forecasting skills of the portfolio manager, and the level of risk aversion of the portfolio manager also affects it. The standard deviation of security returns is $\sigma_r^2 = \sigma_s^2 + \sigma_e^2$, so

$$\beta_x = \frac{\frac{\sigma_s^2}{A\sigma_{e_i}^2}}{\sigma_s^2 + \sigma_e^2} \quad (27)$$

The variance in portfolio weight explained by the informative signal in regression equation (23) is:

$$\beta^2 \sigma_r^2 = \left(\frac{\frac{\sigma_s^2}{A\sigma_{e_i}^2}}{\sigma_r^2} \right)^2 (\sigma_r^2) = \frac{\sigma_s^4}{A^2 \sigma_{e_i}^4} \frac{1}{\sigma_r^2} = \frac{\sigma_s^2}{A^2 \sigma_{e_i}^4} - \frac{\sigma_s^2 (\sigma_{e_i}^2 + \sigma_n^2) / \sigma_r^2}{A^2 \sigma_{e_i}^4}. \quad (28)$$

The explained variance measure in this way is a downward biased estimator of noise trading since the ratio $(\sigma_{e_i}^2 + \sigma_n^2) / \sigma_r^2$ is close to 1 for realistic parameter choices.

The variance of the portfolio weights of the informed noise trader is:

$$\sigma_x^2 = \frac{\sigma_s^2 + \sigma_n^2}{A^2 \sigma_{e_i}^4} \quad (29)$$

And the residual variance represents the variance due to noise trading:

$$\frac{\sigma_s^2 + \sigma_n^2}{A^2 \sigma_{e_i}^4} - \frac{\sigma_s^4}{A^2 \sigma_{e_i}^4} \frac{1}{\sigma_r^2} = \frac{\sigma_n^2}{A^2 \sigma_{e_i}^4} + \frac{\sigma_s^2 ((\sigma_{e_i}^2 + \sigma_n^2) / \sigma_r^2)}{A^2 \sigma_{e_i}^4} \quad (30)$$

Consistent with our objective, the contribution of the residual variance to total noise trading is determined by the size of the noisy signal. However, the second term of residual variance is also affected by the signal, so the residual variance is an upward biased measure of noise trading. These biases are also present in the correlation coefficient:

$$\rho_{xr} = \frac{\sigma_s^2 \left(1 - \frac{\sigma_{e_i}^2}{\sigma_r^2} \right)}{\sigma_s^2 + \sigma_n^2} \quad (31)$$

However, the correlation coefficient has the advantage of being cleaned of the risk aversion parameter. In table 3 we present the outcomes for the analysis based on equation (24).

Table 3: Regression analysis as a means to measure the level of noise trading

	(1) uninformed trader	(2) informed trader	(3) informed trader	(4) informed noise trader	(5) noise trader
Parameters					
p	4.0%	4.0%	4.0%	4.0%	4.0%
sigma r	20.0%	20.0%	20.0%	20.0%	20.0%
sigma s	0.00%	4.00%	2.83%	2.83%	0.00%
sigma n	0.00%	0.00%	0.00%	2.83%	4.00%
sigma e	20.0%	19.6%	19.8%	19.6%	19.6%
A	2	2	2	2	2
Estimators					
β_x	0.0%	52.1%	25.5%	26.0%	0.0%
Explained var	0.0%	1.1%	0.3%	0.3%	0.0%
Resid var	0.0%	26.0%	12.8%	26.9%	27.1%
Total var	0.00%	27.13%	13.02%	27.13%	27.13%
ρ_{xr}	N/A	4.0%	2.0%	2.0%	0.0%

This table illustrates that this approach is not really useful for estimating the level of noise trading in the portfolio. The residual variance from equation (24) is much higher than can be explained by the actual level of noise trading, which would lead us to find no residual variance for the informed traders and a residual variance for the informed noise trader equal to .3%. The reason is of course that we explain the changes in portfolio weights from the realized return, which not only involves the informative signal.

4.3 The noise-corrected signal measure

In section 4.1 we showed that the signal measure underestimates the size of the informative signal in the presence of a noisy signal, and the degree of underestimation increases with the level of noise. In Plantinga [1999] we developed a performance measure that tries to overcome this problem of measuring the informative signal in the presence of noisy signals. We label this measure as the ‘noise corrected signal measure’, and it is defined as:

$$NS \equiv \sqrt{\frac{\sigma_{xr}}{\sqrt{\frac{\sigma_x^2}{\sigma_s^2 + \sigma_n^2}}}} = \sigma_s \quad (32)$$

This definition of the noise-corrected signal measure is consistent with the Dybvig and Ross model. However, the noise-corrected signal measure is not defined if the covariance between asset weights and returns is negative. Therefore we propose the following version of the ‘noise-corrected’ signal measure:

$$NS^* \equiv \iota \frac{|\sigma_{xr}|}{\sqrt{\frac{\sigma_x^2}{\sigma_s^2 + \sigma_n^2}}}, \quad (33)$$

where $\iota = 1$ if $\sigma_{xr} \geq 0$ and $\iota = -1$ if $\sigma_{xr} < 0$.

With observable portfolio weights, both the covariance between weights and returns as well as the variance of portfolio weights can be estimated directly from the data. However, the sum of the variance of informative and the noisy signal is difficult to measure. Our solution is to measure the variance of the perceived signal as follows. Define a new variable q_x , which is the portfolio weight divided by the average portfolio weight:

$$q \equiv \frac{x}{E[x]} = \frac{\frac{p+s+n}{A\sigma_e^2}}{\frac{p}{\sigma_e^2}} = \frac{p+s+n}{p} \quad (34)$$

Calculate the standard deviation of this variable, which is equal to:

$$\sigma_q = \frac{1}{p} \sqrt{\sigma_s^2 + \sigma_n^2} \quad (35)$$

Multiplication of the variance of q with the risk premium results in an estimate of the total of the signals received by the manager, which can be used in equation (33) to get an unbiased estimate of the standard deviation of the informative signal. The noise component is the complement:

$$\sigma_n^2 = p^2 \sigma_q^2 - \sigma_s^2.$$

Applying this methodology to the examples presented in the earlier sections would yield exact estimates of the informative and the noisy signals. However, in empirical applications of this measure, the quality of the estimates will critically depend on the accuracy of the estimation of the risk premium that is used to derive the standard deviation of the total of the signals received by the portfolio manager.

This is illustrated in table 4, where we first present the outcome of the noise corrected signal measure NS with the correct risk premium of 4%. Next we present the

outcomes of the *NS* with the incorrect risk premium of 6%, which leads us to believe that the manager acts on far bigger signals than he actually does. As a result, we are tempted to believe that all managers act on noise, even for those managers that trade on informative signals only.

	(1) uninformed trader	(2) informed trader	(3) informed trader	(4) informed noise trader	(5) noise trader
σ_q	0.00%	100.00%	70.71%	100.00%	100.00%
Correct risk premium					
Total signal	0.0%	4.00%	2.83%	4.00%	4.00%
<i>NS</i>	N/A	4.00%	2.83%	2.83%	4.00%
σ_n	N/A	0.00%	0.00%	2.83%	
Risk premium equal to 6%					
Total signal	0.00%	6.00%	4.24%	6.00%	6.00%
<i>NS</i>	N/A	4.90%	3.46%	3.46%	0.00%
σ_n	N/A	3.46%	2.45%	4.90%	6.00%

5. Discussion

In this paper, we started our analysis with the model of Dybvig and Ross [1985]. In this model an informed portfolio manager acts on an informative signal without noise. They used this model to show that Jensen's alpha is negatively affected by the presence of the informative signal that eventually can lead to a failure in the identification of a successful portfolio manager. Elton and Gruber [1993] proposed to use the covariance between portfolio weights and subsequent returns to overcome this problem. We showed that the standard deviation of portfolio weights could also be used for the purpose of identifying successful portfolio managers. However, in the likely case of portfolio managers acting both on informative and noisy signals, the standard deviation is no longer sufficient. Therefore, we extended the model of Dybvig and Ross with noisy signals and analyzed the consequences of noisy signals for measuring portfolio performance. We showed that the main effect of noisy signals is due to the overconfidence of the portfolio manager in his own forecast, which result on average in higher allocations to risky assets. Since most performance measures are focused on the measurement of the signal, the noise-trading component is ignored. Consequently, the residual variance found in estimating Jensen's alpha is usually attributed to the lack of diversification with a portfolio. However, this residual variance can be partly attributed to noise trading as well. Therefore, we propose to measure noise trading separately using two methods. The first method is to explain the current allocation to risky assets with its future return realization using linear regression. This method contains a slight bias that tends to overestimate the degree of noise trading. The second method called the noise-corrected signal measure overcomes this problem. This second measure critically relies on an adequate estimate

of the risk premium. However, with accurate estimates of the risk premium we believe that this method may help us to estimate the level of noise trading in investment portfolios.

Literature

Black, F., 1986, 'Noise', *Journal of Finance*, Vol. XLI, No.3, pp. 529-543.

DeLong, J.B., A. Shleifer, L.H. Summers, and R.J. Waldmann, 1990, 'Noise trader risk in financial markets', *Journal of Political Economy*, Vol. 98, No. 4, pp. 703-738.

Dybvig, P.H. en S.A. Ross, 1985, 'Differential information and performance measurement using a security market line', *Journal of Finance*, Vol. XL., N. 2, pp. 383-399.

Jensen, M., 1968, 'The performance of mutual funds in the period 1945-64', *Journal of Finance*, Vol. XXIII, No. 2, pp. 389-416.

Grinblatt and Titman, 1989, 'Portfolio Performance Evaluation: Old Issues and New Insights', *Review of Financial Studies*, Vol 2, No.3, pp. 393-421.

Grinblatt and Titman, 1993, 'Performance Measurement without Benchmarks: An Examination of Mutual Fund Returns', *Journal of Business*, Vol. 66, No. 1, pp. 47-68.

Appendix

There are no fund managers who report on the standard deviation of portfolio weights. However, most portfolio managers report the turnover in their portfolio, which is the sum of all transactions as a fraction of total portfolio value. In a static model, turnover is rather meaningless since the portfolio is bought at the start of the period, which means that in the practitioners' definition turnover is equal to the value of the securities in the portfolio. However, we also could interpret our model as a dynamic model by imposing the assumption of stationarity on the stochastic variables. Now, turnover is the expected value of the absolute change in portfolio composition:

$$E[TO] = E \left[\left| \frac{s_{t+1}}{A\sigma_e^2} - \frac{s_t}{A\sigma_e^2} \right| \right]$$

The standard deviation of portfolio weights is related to the expected value of portfolio turnover: as value

$$E[TO] = 2 \frac{\sigma_x}{\sqrt{\pi}} = 2 \frac{\sqrt{\sigma_s^2 + \sigma_n^2}}{A\sigma_e^2 \sqrt{\pi}}$$

We now have a relation between the turnover in a portfolio in the manager's signal structure.